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AN EOQ INVENTORY MODEL FOR DETERIORATING ITEMS WITH QUADRATIC DEMAND AND PARTIAL BACKLOGGING

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Abstract In this paper, an economic order quantity (EOQ) model for deteriorating items with quadratic demand rate and partial backlogging is considered. Shortages are allowed. The deterioration rate is assumed to be constant and the demand rate is the quadratic function of time. The backlogging rate is variable and dependent on the waiting time for the next replenishment. The purpose of the study is to find the optimal policy for minimizing the total inventory cost. The results are illustrated by a numerical example. Sensitivity analysis of various parameters is carried out. Justification for considering time quadratic demand is discussed.

Keywords: Deteriorating Items; EOQ; Inventory; Partial Backlogging; Quadratic Demand

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1. Introduction

In the real world problems, several factors play important roles on controlling inventory for deteriorating items. Over the last three decades, several researchers have studied inventory problems for deteriorating items such as volatile liquids, medicines, blood banks, electronic components and fashion goods. According to the study of Wee [19], deteriorating items refers to the items that become decayed, damaged, spoilage, expired, evaporative, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become damaged, decayed, evaporated or expired through time like fish, meat, vegetables, fruits, flowers, medicines, films and so on where as the other category refers to the items that lose part or total value through time because of new technology or the introduction of new alternatives like computer chips, mobile phones, fashion and seasonal goods and so on. The items of first category have the short natural life cycles whereas those of second category have the short market life cycle. Also, in case of the second category, after a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons.

The inventory problem for deteriorating items was first studied by Whitin [21] on fashion items. He found fashion items deteriorating at the end of the storage period. Then Ghare and Schrader [4] developed a model for an exponential decaying inventory. According to them, the consumption of the deteriorating items was closely relative to a negative exponential function of time. Shah and Jaiswal [15] and Aggarwal [1] presented and reestablished an order level inventory model with a constant rate of deterioration respectively. An inventory model for deteriorating items with time proportional demand when shortages were prohibited was developed by Dave and Patel [3]. Later, Sachan [14] extended the model to allow for shortages. This inventory model laid foundations for the follow-up study. Raafat [13], Goyal and Giri [6] and Li et al. [10] made the comprehensive literature reviews on deteriorating items in 1991, 2001 and 2010 respectively. Hollier and Mak [8] studied the optimal replenishment policies under both constant and variable replenishment intervals. Hariga and Benkherouf [7] developed the model for deteriorating items by taking exponentially demand. Singh and Pattnayak [16] studied the inventory model for deteriorating items with trapezoidal type demand rate and weibull distribution deterioration rate.

For the review literature, it is clear that researchers have paid their attention on only two types of time dependent demand, namely linear and exponential. Linear demand shows steady increases (or decreases) in demand which may be rarely seen to occur in the real market. For other case, demand is unlikely to increase at a rate which is as high as exponential. In order to avoid such difficulties, quadratic demand rate seems to be a better representation in the real market situations. Khanra and Chaudhuri [9] and Ghosh and Chaudhuri [5] established their models by considering the demand rate as quadratic function of time. Singh and Pattnayak [18] developed an EOQ model for deteriorating items with quadratic demand under permissible delay in payment. In real life situation shortages are not completely backlogged. Some customers would like to wait for backlogging during the shortage period, but the others would not. Consequently, the opportunity cost due to lost sells should be considering while developing the model. Several researchers such as Wee [20], Pak [12] and Hollier and Mak [8] also considered the backlogging rates in their models. The length of waiting time for the next replenishment is the main factor in calculating the backlogging rate. Therefore, the backlogging rate is variable and dependent on the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be and vice versa. During the shortage period, the backlogging rate is variable and dependent on the length of waiting time for the next replenishment. Chang and Dye [2] developed an EOQ model for deteriorating items considering shortage. Ouyang et al. [11] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. In a recent paper, Singh and Pattnayak [17] developed an EOQ model for deteriorating items with linear demand and partial backlogging.

In this paper, an effort has been made to analyze an EOQ model for deteriorating items considering the demand as the quadratic function of time. Shortages are allowed and partially backlogged. Among the various demands in EOQ models, the more realistic demand approach is to consider a quadratic demand rate because it represents both accelerated and retarded growth in demand. Accelerated growth is found to occur in the case of spare parts of newly introduced the state-of-the-art aircrafts, computers etc. On the other hand, the retarded growth is found to be occur in the case of spare parts of the obsolete aircrafts, computers etc. The demand rate in these cases is a quadratic function of time which is of the form $R(t) = a + bt + ct^2$. Here $c = 0$ represents the linear demand rate and $b = c = 0$ represents constant demand rate. In this model, we assume the demand rate as a quadratic function of time and the backlogging rate is inversely proportional to the waiting time for the next replenishment. The objective of the problem is to minimize the total relevant cost by simultaneously optimizing the shortage point and the length of the cycle. The model is illustrated with a numerical example. Also the sensitivity analysis of the model is examined for changes in the parameters.

2. ASSUMPTIONS

The following assumptions are made in developing the model.

(i) The demand rate for the item is represented by a quadratic function of time.

(ii) The deterioration rate is constant on the on hand inventory per unit time and there is no repair or replenishment of the deteriorated items within the cycle.

(iii) Replenishment rate is infinite, i. e. replenishment is instantaneous.

(iv) The inventory system involves only one item and the planning horizon is infinite.

(v) Shortages are allowed.

(vi) During the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Hence, the proportion of customers who would like to accept backlogging at time t is decreasing with the waiting time $(T-t)$ waiting for the next replenishment. To take care of this situation we have defined the backlogging rate to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant, $t_1 \leq t \leq T$.

3. NOTATIONS

The following notations have been used in developing the model.

(i) The demand rate is $R(t) = \begin{cases} a+bt+ct^2, & I(t) > 0; \\ D_0, & I(t) \geq 0. \end{cases}$ where $a > 0, b \neq 0$ & $c \neq 0$. Here a is initial demand rate and b is the increasing demand rate. The rate of change in the demand rate itself changes at a rate c .

(ii) The deteriorating rate $\theta(t) = \theta, 0 < \theta \ll 1$ is the constant rate of deterioration of an item.

(iii) C_1 : holding cost, \$/per unit/per unit time.

(iv) C_2 : cost of the inventory item, \$/per unit.

(v) C_3 : ordering cost of inventory, \$/per order.

(vi) C_4 : shortage cost, \$/per unit/per unit time.

(vii) C_5 : opportunity cost due to lost sales, \$/per unit.

(viii) t_1 : time at which shortages start.

(ix) T : length of each ordering cycle.

(x) W : the maximum inventory level for each ordering cycle.

(xi) S : the maximum amount of demand backlogged for each ordering cycle.

(xii) Q : the economic order quantity for each ordering cycle.

(xiii) $I(t)$: the inventory level at time t .

(xiv) t_1^* : the optimal solution of t_1.

(xv) T^* : the optimal solution of T .

(xvi) Q^* : the optimal economic order quantity.

(xvii) W^* : the optimal maximum inventory level.

(xviii) TC^* : the minimum average total cost per unit time.

4. MATHEMATICAL FORMULATION OF THE MODEL

We consider the deteriorating inventory model with quadratic demand rate. Replenishment occurs at time $t = 0$ when the inventory level attains its maximum, W. From $t = 0$ to t_1 , the inventory level reduces due to demand and deterioration. At time t_1, the inventory level achieves zero, then shortage is allowed to occur during the time interval $[t_1, T]$ and all of the demand during shortage period $[t_1, T]$ is partially backlogged.

As the inventory level reduces due to quadratic demand rate as well as deterioration during the inventory interval $[t_1, T]$, the differential equation representing the inventory status is governed by

$$\frac{dI(t)}{dt} + \theta(t)\, I(t) = -R(t), 0 \le t \le t_1. \tag{1}$$

where $\theta(t) = \theta$ and $R(t) = a + bt + ct^2$.

The solution of Eq. (1) using the condition $I(t_1) = 0$ is

$$I(t) = \left(\frac{a + bt_1 + ct_1^2}{\theta} - \frac{b + 2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right) e^{\theta(t_1 - t)}$$

$$-\frac{a + bt + ct^2}{\theta} + \frac{b + 2ct}{\theta^2} - \frac{2c}{\theta^3}, 0 \le t \le t_1. \tag{2}$$

If $c = 0$, then Eq. (2) represents the instantaneous inventory level at any time t for linear demand rate. Also, putting $b = c = 0$ in Eq. (2), we get the instantaneous inventory level at time t for constant demand rate.

Maximum inventory level for each cycle is obtained by putting the boundary condition $I(0) = W$ in Eq. (2). Therefore,

$$W = I(0) = \left(\frac{a + bt_1 + ct_1^2}{\theta} - \frac{b + 2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right) e^{\theta t_1} - \frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}. \tag{3}$$

During the shortage interval $[t_1, T]$, the demand at time t is partially backlogged at the fraction $\frac{1}{1+\delta(T-t)}$. Therefore, the differential equation governing the amount of demand backlogged is

$$\frac{dI(t)}{dt} = -\frac{D_0}{1 + \delta(T - t)}, t_1 < t \le T. \tag{4}$$

with the boundary condition $I(t_1) = 0$.

The solution of Eq. (4) is

$$I(t) = \frac{D_0}{\delta}\left[\ln\{1 + \delta(T - t)\} - \ln\{1 + \delta(T - t_1)\}\right], t_1 < t \le T. \tag{5}$$

Maximum amount of demand backlogged per cycle is obtained by putting $t = T$ in Eq. (5). Therefore,

$$S = -I(T) = \frac{D_0}{\delta}\ln\left[1+\delta(T-t_1)\right]. \tag{6}$$

Hence, the economic order quantity per cycle is

$$Q = W + S = \left(\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right)e^{\theta t_1}$$

$$-\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} + \frac{D_0}{\delta}\ln\left[1+\delta(T-t_1)\right]. \tag{7}$$

The inventory holding cost per cycle is

$$HC = C_1\int_0^{t_1}\left[\begin{array}{l}\left(\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right)e^{\theta(t_1-t)} \\ -\frac{a+bt+ct^2}{\theta} + \frac{b+2ct}{\theta^2} - \frac{2c}{\theta^3}\end{array}\right]dt$$

$$= C_1\left[\begin{array}{l}\left(\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right)\left(\frac{e^{\theta t_1}-1}{\theta}\right) \\ -\frac{1}{\theta}\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3}\right) + \frac{1}{\theta^2}\left(bt_1 + ct_1^2\right) - \frac{2ct_1}{\theta^3}\end{array}\right]. \tag{8}$$

The deterioration cost per cycle is

$$DC = C_2\left[W - \int_0^{t_1}\left(a+bt+ct^2\right)\right]$$

$$= C_2\left[\begin{array}{l}\left(\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right)e^{\theta t_1} \\ -\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} - at_1 - \frac{bt_1^2}{2} - \frac{ct_1^3}{3}\end{array}\right]. \tag{9}$$

The shortage cost per cycle is

$$SC = C_4\left[-\int_{t_1}^{T}\left[\frac{D_0}{\delta}\left[\ln\left\{1+\delta(T-t)\right\} - \ln\left\{1+\delta(T-t_1)\right\}\right]\right]dt\right]$$

$$= \frac{C_4 D_0}{\delta}\left[(T-t_1) - \frac{1}{\delta}\ln\left\{1+\delta(T-t_1)\right\}\right]. \tag{10}$$

The opportunity cost due to lost sales per cycle is

$$OC = C_5\int_{t_1}^{T}\left[D_0\left(1 - \frac{1}{1+\delta(T-t)}\right)\right]dt$$

$$= C_0 D_5\left[(T-t_1) - \frac{1}{\delta}\ln\left\{1+\delta(T-t_1)\right\}\right]. \tag{11}$$

Therefore, the average total cost per unit time per cycle = (holding cost + deterioration cost + ordering cost + shortage cost + opportunity cost due to lost sales) / length of the ordering cycle, i.e.,

$$TC = TC\left(t_1, T\right) = \frac{C_1}{T}\left[\begin{array}{l}\left(\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right)\left(\frac{e^{\theta t_1}-1}{\theta}\right) \\ -\frac{1}{\theta}\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3}\right) + \frac{1}{\theta^2}\left(bt_1 + ct_1^2\right) - \frac{2ct_1}{\theta^3}\end{array}\right]$$

$$+\frac{C_2}{T}\left[\left(\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right)e^{\theta t_1} - \frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} - at_1 - \frac{bt_1^2}{2} - \frac{ct_1^3}{3}\right]$$

$$+\frac{C_3}{T} + \frac{D_0\left(C_4 + \delta C_5\right)}{\delta T}\left[\left(T - t_1\right) - \frac{1}{\delta}\ln\left\{1 + \delta\left(T - t_1\right)\right\}\right]. \qquad (12)$$

Our aim is to determine the optimal values of $t_1$and T in order to minimize the average total cost per unit time, TC .

Using calculus, we now minimize TC . The optimum values of t_1 and $T$for the minimum average cost TC are the solutions of the equations

$$\frac{\partial\left(TC\right)}{\partial t_1} = 0 \ \ and \ \ \frac{\partial\left(TC\right)}{\partial T} = 0. \qquad (13)$$

provided that they satisfy the sufficient conditions $\frac{\partial^2(TC)}{\partial t_1^2} > 0, \frac{\partial^2(TC)}{\partial T^2} > 0$ and $\frac{\partial^2(TC)}{\partial t_1^2}.\frac{\partial^2(TC)}{\partial T^2} - \left(\frac{\partial^2(TC)}{\partial t_1 \partial T}\right)^2 > 0.$

Eq. (13) can be written as

$$\frac{C_1}{T}\left\{\begin{array}{l}\left(\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right)e^{\theta t_1} + \left(\frac{b+2ct_1}{\theta^2} - \frac{2c}{\theta^3}\right)\left(e^{\theta t_1} - 1\right) \\ -\frac{1}{\theta}\left(a + bt_1 + ct_1^2\right) + \frac{1}{\theta^2}\left(b + 2ct_1\right) - \frac{2c}{\theta^3}\end{array}\right\}$$

$$+\frac{C_2}{T}\left(a + bt_1 + ct_1^2\right)\left(e^{\theta t_1} - 1\right) - \frac{1}{T}\left[\frac{D_0\left(C_4 + \delta C_5\right)\left(T - t_1\right)}{1 + \delta\left(T - t_1\right)}\right] = 0. \qquad (14)$$

and

$$\frac{1}{T}\left[\frac{D_0\left(C_4 + \delta C_5\right)\left(T - t_1\right)}{1 + \delta\left(T - t_1\right)} - \left(TC\right)\right] = 0. \qquad (15)$$

Now, $t_1^*$and T^* are obtained from the Eq.s (13) and (14) respectively. Next, by using $t_1^*$and T^*, we can obtained the optimal economic order quantity, the optimal maximum inventory level and the minimum average total cost per unit time from Eq.s (7), (3) and (12) respectively.

5. NUMERICAL EXAMPLES

Numerical Example

In this section, we provide a numerical example to illustrate the above theory.

Example 1: Let us take the parameter values of the inventory system as follows: $a = 12$, $b = 2$, $c = 2$, $C_1 = 0.5$, $C_2 = 1.5$, $C_3 = 3$, $C_4 = 2.5$, $C_5 = 2$, $D_0 = 8$, $\theta = 0.01$and $\delta = 2$.

Solving Eq.s (14) and (15), we have the optimal shortage period $t_1^* = 0.789981$ unit time and the optimal length of ordering cycle $T^* = 0.941824$ unit time. Thereafter, we get the optimal order quantity $Q^* = 11.5361$ units, the optimal maximum inventory level $W^* = 10.4753$ units and the minimum average total cost per unit time $TC^* = 6.05656$.

6. SENSITIVITY ANALYSIS

We study now study the effects of changes in the values of the system parameters a , b, c , C_1 , C_2, C_3, C_4, C_5, D_0, θ and δ on the optimal total cost and number of reorder. The sensitivity analysis is performed by changing each of parameters by +50%, +25%, -25% and -50% taking one parameter at a time and keeping the remaining parameters unchanged.

The analysis is based on the Example 1 and the results are shown in **Table 1**. The following points are observed.

(i) t_1^* & T^*decrease while Q^*& TC^* increase with the increase in value of the parameter a. Here t_1^*,T^*,Q^*& TC^* are highly sensitivity to change in a.

(ii) t_1^* ,T^*& Q^*decrease while TC^* increases with the increase in value of the parameter b. Here t_1^*,T^*,Q^*& TC^* are moderately sensitivity to change in b.

(iii) t_1^* ,T^*& Q^*decrease while TC^* increases with the increase in value of the parameter c. Here t_1^*,T^*,Q^*& TC^* are lowly sensitivity to change in c.

(iv) t_1^* ,T^*& Q^*decrease while TC^* increases with the increase in value of the parameter C_1. Here t_1^*,T^*,Q^*& TC^* are highly sensitivity to change in C_1.

(v) t_1^* ,T^*& Q^*decrease while TC^* increases with the increase in value of the parameter C_2. Here t_1^*,T^*,Q^*& TC^* are lowly sensitivity to change in C_2.

(vi) t_1^* ,T^*Q^*& TC^* increase with the increase in value of the parameter C_3. Here t_1^*,T^*,Q^*& TC^* are highly sensitivity to change in C_3.

(vii) t_1^* & TC^*increase while T^*& Q^* decrease with the increase in value of the parameter C_4. Here t_1^*,T^*,Q^*& TC^* are lowly sensitivity to change in C_4.

(viii) t_1^* & TC^*increase while T^*& Q^* decrease with the increase in value of the parameter C_5. Here t_1^*,T^*,Q^*& TC^* are moderately sensitivity to change in C_5.

(ix) t_1^* ,Q^*& TC^*increase while T^* decreases with the increase in value of the parameter D_0. Here t_1^*,T^*,Q^*& TC^* are moderately sensitivity to change in D_0.

(x) t_1^* ,T^*& Q^*decrease while TC^* increases with the increase in value of the parameter θ. Here t_1^*,T^*,Q^*& TC^* are lowly sensitivity to change in θ.

(xi) t_1^* & TC^*increase while T^*& Q^* decrease with the increase in value of the parameter δ. Here t_1^*,T^*,Q^*& TC^* are moderately sensitivity to change in δ.

TABLE 1. Sensitivity analysis

Parameters	% Change in parameter	t_1^*	T^*	% Change in Q^*	% Change in TC^*
$a = 12$	+50	0.666874	0.849958	+0.208	+0.151
	+25	0.723285	0.891306	+0.112	+0.08
	-25	0.869438	1.00387	-0.132	-0.09
	-50	0.964379	1.08006	-0.29	-0.193
$b = 2$	+50	0.767626	0.922332	-0.002	+0.014
	+25	0.778475	0.93177	-0.001	+0.007
	-25	0.80222	0.952568	+0.001	-0.008
	-50	0.815282	0.964088	+0.002	-0.015
$c = 2$	+50	0.770924	0.924449	-0.01	+0.008
	+25	0.780097	0.932797	-0.005	+0.004
	-25	0.800692	0.951644	+0.006	-0.005
	-50	0.81238	0.962403	+0.013	-0.009
$C_1 = 0.5$	+50	0.647963	0.833244	-0.162	+0.161
	+25	0.710032	0.879441	-0.093	+0.086
	-25	0.899421	1.03148	+0.134	-0.103
	-50	1.06544	1.17449	+0.352	-0.231
$C_2 = 1.5$	+50	0.784511	0.937464	-0.006	+0.006
	+25	0.787233	0.939632	-0.003	+0.003
	-25	0.792754	0.944039	+0.003	-0.003
	-50	0.795553	0.946278	+0.007	-0.006

Parameters	% Change in parameter	t_1^*	T^*	% Change in Q^*	% Change in TC^*
$C_3 = 3$	+50	0.930025	1.1328	+0.212	+0.239
	+25	0.864992	1.04251	+0.112	+0.125
	-25	0.700665	0.825914	-0.128	-0.14
	-50	0.58855	0.685329	-0.283	-0.304
$C_4 = 2.5$	+50	0.798655	0.922194	-0.004	+0.014
	+25	0.794715	0.930962	-0.002	+0.008
	-25	0.784189	0.955613	+0.003	-0.009
	-50	0.776944	0.973662	+0.006	-0.021
$C_5 = 2$	+50	0.80259	0.913678	-0.006	+0.021
	+25	0.797161	0.92549	-0.003	+0.012
	-25	0.780049	0.965815	+0.005	-0.016
	-50	0.765444	1.00424	+0.012	-0.039
$D_0 = 8$	+50	0.807758	0.90285	+0.022	+0.029
	+25	0.800717	0.917702	+0.013	+0.017
	-25	0.771658	0.987417	-0.022	-0.029
	-50	0.733728	1.10225	-0.068	-0.089
$\theta = 0.01$	+50	0.783621	0.93667	-0.006	+0.006
	+25	0.786786	0.939236	-0.003	+0.003
	-25	0.793207	0.944441	+0.003	-0.003
	-50	0.796465	0.947088	+0.006	-0.006
$\delta = 2$	+50	0.79986	0.924080	-0.006	+0.016
	+25	0.795455	0.932077	-0.003	+0.009
	-25	0.782984	0.953986	+0.005	-0.011
	-50	0.773699	0.969627	+0.011	-0.026

7. CONCLUSIONS

In the classical EOQ model, the demand rate of an item was assumed as constant. Shortages were not allowed. However, in the real market, the demand rate of any product is always in a dynamic state. In reality, models are developed not only considering the variation of demand rate with time, but also the costs are affected by shortages the proposed model is based on the quadratic demand rate. While developing the model the researchers usually take the demand rate to be either a linear demand rate or an exponential demand rate. The linear demand rate is of the form $R(t) = a + bt$, $a \geq 0$, $b \neq 0$ which implies the steady increase when $b > 0$ or decrease when $b < 0$. These cases are rarely seen to occur for any product. On the other hand, an exponential demand rate is of the form $R(t) = ae^{bt}$, $a > 0$, $b \neq 0$which implies an exponential increase when $b > 0$or decrease when $b < 0$. Due to high exponential rate, it is doubtful whether the real market demand of any product can really rise or fall exponentially. Therefore a better alternative is either accelerated rise or fall in demand. Accelerated growth in demand rate takes place in case of the state-of-the-art aircrafts, super computers, machines and there spare parts. On the other hand the accelerated decline in the demand rate is considered to occur in the case of obsolete aircrafts, supercomputers, machines and their spare parts. The demand rate of such type of seasonal product of rises rapidly to a peak in the mid season and then falls rapidly when the season wanes out. These different cases of demand rate can be represented by the form $R(t) = a + bt + ct^2$, $a \geq 0$, $b \neq 0$, $c \neq 0$. Here $\frac{dR(t)}{dt} = b + 2ct \,\&\, \frac{d^2R(t)}{dt^2} = 2c$.

Case I: When $b > 0$, $c > 0$, the demand rate $R(t)$increases which is called the accelerated growth in demand.

Case II: When $b > 0$, $c < 0$, the demand rate is called the retarded growth in demand rate.

Case III: When $b < 0$, $c > 0$, the demand rate is called the accelerated decline in demand rate.

Case IV: When $b < 0$, $c < 0$, the demand rate is called the retarded decline in demand rate.

This advantage of quadratic demand rate has motivated authors to consider in the proposed model. Here the deterioration rate is taken to be constant and the backlogging rate is inversely proportional to the waiting time for the next replenishment. Finally, the optimal solution is obtained by minimizing the total inventory cost function.

The proposed model can be extended in numerous ways. For example, we may extend the quadratic demand to more generalized demand pattern and stochastic demand pattern. Also, we could consider the variable deterioration rate such as two-parameter weibull distribution deterioration and three- parameter weibull distribution deterioration and gamma distribution. Finally we could generalize the model to allow for an economic production lot size model.

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