

**DIFFUSION-THERMO EFFECT ON FREE CONVECTIVE
FLOW OF VISCO-ELASTIC FLUID IN A VERTICAL
CHANNEL**

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Abstract :An analysis is made to study the effect of fully developed free convection and mass transfer on the flow of visco-elastic fluid in a vertical channel formed by two vertical parallel plates under the influence of asymmetric wall temperature and concentration. The diffusion-thermo effect renders the present analysis interesting and curious. The Laplace transform technique has been applied to solve equations governing the flow phenomenon.

The result of present study has been compared with the previous findings reported without elasticity, mass transfer and diffusion thermo effect. The validity of our result is assured on the fact that our result is good agreement with the previous authors. The study also revealed that the steady state of the problem is independent of the Dufour effect.

Keywords:Visco-elastic, Diffusion thermo effect, mass flux, sustentation.

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1. Introduction

The study of motions of fluids is one of the most successful and useful applications of mathematics. In classical viscous fluid we know that the fluid exerts a viscosity effect when there is a tendency for shear flow or tangential

flow of the fluid. The other types of fluids called visco-elastic fluids, which possess a certain degree of elasticity in addition to their viscosity. These visco-elastic fluids in the course of their motion store up energy in the material as strain energy. The remaining energy is lost due to viscous dissipation.

Fourier's law, for instance, described the relation between energy flux and temperature gradient. In other aspect, Fick's law was determined by the correlation of mass flux and concentration gradient. Moreover, it was found that energy flux can also be generated by composition gradients, pressure gradients, or body forces. The energy flux caused by a composition gradient was discovered in 1873 by Dufour and was correspondingly referred to the Dufour effect. It was also called the diffusion-thermo effect. On the other hand, mass flux can also be created by a temperature gradient, as was established by Soret. This is the thermal-diffusion effect. In general, the thermal-diffusion and the diffusion-thermo effects were of a smaller order of magnitude than the effects described by Fourier's or Fick's law and were often neglected in heat mass transfer processes. There were still some exceptional conditions. The thermal-diffusion effect has been utilized for isotope separation and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air), the diffusion-thermo effect was found to be of a magnitude such that it may not be neglected in certain conditions.

In nature, flow occurs due to density differences caused by temperature as well as chemical composition gradients. Therefore, it warrants the simultaneous consideration of temperature difference as well as concentration difference when heat and mass transfer occurs simultaneously. It has been found that an energy flux can be created not only by temperature gradients but by composition gradients also. This is called Dufour effect. If, on the other hand, mass fluxes are created by temperature gradients, it is called the Soret effect.

Jha andajibade [1] have studied free convection heat and mass transfer flow in a vertical chanel with the Dufour Effect. Singh et al. [2] studied the transient free convection flow between two vertical parallel plates. In view of importance of the diffusion-thermo effect Kafoussias and Williams [3] have studied the effects of thermal diffusion and diffusion-thermo effect on mixed free and forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Dursunkaya and Worek [4] have studied the diffusion-thermo and thermal diffusion effects in transient and steady natural convections from a vertical surface.

Aberu et al. [5] have studied the boundary layer flows with Dufour and Soret effects. Osalusi et al. [6] have worked on mixed and free convective heat and mass transfer of an electrically conducting fluid considering Dufour and Soret effects. Other notable studies on the Soret and the Dufour effects on heat and mass transfer are the works of Kafoussias [7], Anghel et al. [8], Postelnick [9], Alam et al. [10,11] and Alam and Rahaman [12].

Chambra and Yang [13] have worked on thermal diffusion of a chemically reactive species in a laminar boundary layer flow. References [14, 15] focused on the study of convective heat and mass transfer incompressible viscous Boussinesq fluid in the presence of chemical reaction of first order. References [16, 17] discussed the effects of thermo-diffusion (Soret effects) and diffusion-thermo (Dufour effects) on MHD mixed convection heat and mass transfer of an electrically conducting fluid. Sharma et al. [18] discussed the unsteady MHD free convection heat and mass transfer of viscous fluid flowing through a Darcian porous regime adjacent to a moving vertical semi-infinite plate under Soret and Dufour effects.

The ratio of concentration to thermal buoyancy is called the sustentation parameter (N) whose positive values mean that the convections due to heat and mass transfer support each other and negative values denote the converse.

Recently, numerical solutions of chemically reacting mixed convective heat and mass transfer along inclined and vertical plates with Soret and Dufour effects were studied by Beg et al. [19,20] by concluding that skin friction increases with positive increase in β . The unsteady MHD heat and mass transfer free convection flow of a polar fluid past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion have studied by Saxena and Dubey [21]. Raveendra Babu et al. [22] studied diffusion-thermo and radiation effects on MHD free convective heat and mass transfer flow past an infinite vertical plate in the presence of a chemical reaction of first order. Sudhakar et al [23] discussed chemical reaction effect on an unsteady MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo.

The objective of the present study is to consider the effect of non-Newtonian parameter particularly elastic parameter on the flow characteristics. The flow of non-Newtonian flow model (Walters) has been considered in a vertical channel with uniform temperature and concentration. The consideration of Soret and Dufour effect has resulted in coupling the concentration and energy equations.

2. Mathematical Formulation

Consider the free convective and mass transfer flow of a visco-elastic fluid in a vertical channel formed by two infinite vertical parallel plates. The convective current is induced by both the temperature and concentration gradient. The flow is assumed to be in the x' - direction, which is taken to be vertically upward direction along the channel walls and the y' -axis is taken to be normal to the plate that are h distance apart. At the time $t \leq 0$, the fluid is at rest with the initial temperature T_0 and concentration C_0 .

For $t > 0$, the temperature and concentration on the wall $y'=0$ increase and decrease to T_w and C_w respectively, while the wall $y'=h$ is maintained at T_0 and C_0 . The velocity of the fluid on both walls remains $u'=0$. The governing equations the temperature is governed by concentration, leading to the diffusion -thermo effect . Under the usual Boussinesq's approximations, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_0) + g\beta^*(C' - C_0) - \frac{\kappa_0}{\rho} \frac{\partial^3 u'}{\partial y'^2 \partial t'} \quad (1)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (2)$$

$$\frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y'^2} + D_1 \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

where $\nu, g, \beta, \beta^*, D, \alpha$ and D_1 are kinematic viscosity, acceleration due to gravity, coefficient of thermal expansion, coefficient of mass expansion, chemical molecular diffusivity, the thermal diffusivity and dimensional coefficient of the diffusion thermo effect respectively.

With the following boundary conditions

$$\begin{cases} t' \leq 0 : u'(y', t') = 0, T'(y', t') = T_0, C'(y', t') = C_0 \text{ for all } y \\ t' > 0 : u'(0, t') = 0, T'(0, t') = T_w, C'(0, t') = C_w \text{ at } y' = 0 \\ u'(h, t') = 0, T'(h, t') = 0, C'(h, t') = 0 \text{ at } y' = h \end{cases} \quad (4)$$

Let us introduce the following non dimensional quantities

$$\begin{aligned} y &= \frac{y'}{h}, t = \frac{t'\nu}{h^2}, u = \frac{u'\nu}{g\beta h^2(T_w - T_0)}, Pr = \frac{\nu}{\alpha}, \\ Sc &= \frac{\nu}{d}, Rc = \frac{\kappa_0}{\rho h^2}, T = \frac{T' - T_0}{T_w - T_0}, C = \frac{C' - C_0}{C_w - C_0}, \\ N &= \frac{\beta^*(C_w - C_0)}{\beta(T_w - T_0)}, D^* = \frac{D_1(C_w - C_0)}{\alpha(T_w - T_0)}, \end{aligned} \quad (5)$$

The governing equations from (1) to (3) for the flow in dimensionless form are given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + NC + T - R_c \frac{\partial^3 u}{\partial y^2 \partial t} \tag{6}$$

$$S_c \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \tag{7}$$

$$P_r \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + D^* \frac{\partial^2 C}{\partial y^2} \tag{8}$$

$$\begin{cases} t' \leq 0 : u(y, t) = 0, T(y, t) = 0, C(y, t) = 0 \text{ for all } y \\ t' > 0 : u(0, t) = 0, T(0, t) = 1, C(0, t) = 1 \text{ at } y' = 0 \\ u(1, t) = 0, T(1, t) = 0, C(1, t) = 0 \text{ at } y' = h \end{cases} \tag{9}$$

Here P_r is the Prandtl number, which is inversely proportional to the thermal diffusivity (α) of the working fluid, D^* is the Dufour number, which is the coefficient of the concentration energy, S_c is the Schmidt number and N is the sustentation parameter whose positive values means that convections due to heat and mass transfer support each other and negative values denote the converse.

3. Solution of the problem

The solution of the equations (6) to (8) subject to the boundary conditions (9) are obtained by Laplace Transform technique:

Unsteady Case: The solutions (6)-(8) subject to the boundary conditions given by (9) are obtained as follows:

$$C = \sum_{n=0}^{\infty} \left\{ \operatorname{erfc} \left\{ \frac{a_1 \sqrt{S_c}}{2\sqrt{t}} \right\} - \left\{ \operatorname{erfc} \left(\frac{b_1 \sqrt{S_c}}{2\sqrt{t}} \right) \right\} \right\} \quad (10)$$

$$T = (1 - A_1) \sum_{n=0}^{\infty} \left\{ \operatorname{erfc} \left(\frac{a_1 \sqrt{P_r}}{2\sqrt{t}} \right) - \left\{ \operatorname{erfc} \left(\frac{b_1 \sqrt{P_r}}{2\sqrt{t}} \right) \right\} + A_1 C \right\} \quad (11)$$

$$\begin{aligned} u = & (tA_2 + R_c A_7)C + (tA_3 + R_c A_6) \sum_{n=0}^{\infty} \left[\left\{ \operatorname{erfc} \left(\frac{a_1 \sqrt{P_r}}{2\sqrt{t}} \right) - \left\{ \operatorname{erfc} \left(\frac{b_1 \sqrt{P_r}}{2\sqrt{t}} \right) \right\} \right\} \right. \\ & - (tA_4 + R_c A_8) \sum_{n=0}^{\infty} \left\{ \operatorname{erfc} \left(\frac{a_1}{2\sqrt{t}} \right) - \left\{ \operatorname{erfc} \left(\frac{b_1}{2\sqrt{t}} \right) \right\} \right. \\ & + A_9 \sum_{n=0}^{\infty} \left[a_1^2 \left\{ \operatorname{erfc} \left(\frac{a_1 \sqrt{S_c}}{2\sqrt{t}} \right) - b_1^2 \left\{ \operatorname{erfc} \left(\frac{b_1 \sqrt{S_c}}{2\sqrt{t}} \right) \right\} \right] \\ & + A_{10} \sum_{n=0}^{\infty} \left[a_1^2 \left\{ \operatorname{erfc} \left(\frac{a_1 \sqrt{P_r}}{2\sqrt{t}} \right) - b_1^2 \left\{ \operatorname{erfc} \left(\frac{b_1 \sqrt{P_r}}{2\sqrt{t}} \right) \right\} \right] \\ & - A_5 \sum_{n=0}^{\infty} \left[a_1^2 \left\{ \operatorname{erfc} \left(\frac{a_1}{2\sqrt{t}} \right) - b_1^2 \left\{ \operatorname{erfc} \left(\frac{b_1}{2\sqrt{t}} \right) \right\} \right] \\ & + A_{11} \sqrt{t} \sum_{n=0}^{\infty} \left[a_1 \operatorname{erfc} \left(\frac{-a_1^2 S_c}{2\sqrt{t}} \right) - b_1^2 \operatorname{erfc} \left(\frac{-b_1^2 S_c}{2\sqrt{t}} \right) \right] \\ & + A_{12} \sqrt{t} \sum_{n=0}^{\infty} \left[a_1 \exp \left(\frac{-a_1^2 P_r}{2\sqrt{t}} \right) - b_1^2 \operatorname{erfc} \left(\frac{-b_1^2 P_r}{2\sqrt{t}} \right) \right] \\ & - \frac{R_c A_{11}}{\sqrt{t}} \sum_{n=0}^{\infty} n \left[\exp \left(\frac{-a_1^2}{2\sqrt{t}} \right) + \exp \left(\frac{-b_1^2}{2\sqrt{t}} \right) \right] \\ & - \frac{R_c A_{14}}{\sqrt{t}} \sum_{n=0}^{\infty} (n+1) \left[\exp \left(\frac{-a_1^2}{2\sqrt{t}} \right) + \exp \left(\frac{-b_1^2}{2\sqrt{t}} \right) \right] \end{aligned} \quad (12)$$

where $a_1=2n+y$, $b_1=2n+2-y$.

The other constants are defined in Appendix.

The rate of mass transfer (Sh), the rate of heat transfer (Nu) and the skin friction (τ) on the walls of the channel are obtained as follows: **Sherwood Number:**

$$Sh_0 = -\frac{\partial C}{\partial y}|_{y=0} = \sqrt{\frac{S_c}{\pi t}} \sum_{n=0}^{\infty} \left[\exp\left(-\frac{n^2 S_c}{t}\right) + \exp\left(-\frac{(n+1)^2 S_c}{t}\right) \right] \quad (13)$$

$$Sh_1 = \frac{\partial C}{\partial y}|_{y=1} = -2\sqrt{\frac{S_c}{\pi t}} \sum_{n=0}^{\infty} \exp\left(-\frac{(2n+1)^2 S_c}{t}\right) \quad (14)$$

Nusselt Number:

$$Nu_0 = -\frac{\partial T}{\partial y}|_{y=0} = (1 - A_1) \sqrt{\frac{P_r}{\pi t}} \sum_{n=0}^{\infty} \left[\exp\left(-\frac{n^2 P_r}{t}\right) + \exp\left(-\frac{(n+1)^2 P_r}{t}\right) \right] + A_1 Sh_0 \quad (15)$$

$$Nu_1 = \frac{\partial T}{\partial y}|_{y=1} = 2(A_1 - 1) \sqrt{\frac{P_r}{\pi t}} \sum_{n=0}^{\infty} \exp\left(-\frac{(2n+1)^2 P_r}{t}\right) + A_1 Sh_1 \quad (16)$$

Skin Friction:

$$\begin{aligned}
\tau_0 = -\frac{\partial u}{\partial y}|_{y=0} = & -(tA_2 + R_cA_7)Sh_0 - (tA_3 + R_cA_6)\sqrt{\frac{P_r}{\pi t}} \sum_{n=0}^{\infty} \left[\exp\left(-\frac{n^2 P_r}{t}\right) + \exp\left(-\frac{(n+1)^2 P_r}{t}\right) \right] \\
& + \frac{(tA_4 + R_cA_8)}{\sqrt{\pi t}} \sum_{n=0}^{\infty} \left[\exp\left(-\frac{n^2}{t}\right) + \exp\left(-\frac{(n+1)^2}{t}\right) \right] \\
& + 4A_9 \sum_{n=0}^{\infty} \left[\operatorname{nerfc}\left(\frac{n\sqrt{S_c}}{\sqrt{t}}\right) + (n+1)\operatorname{erfc}\left(\frac{(n+1)\sqrt{S_c}}{\sqrt{t}}\right) \right] \\
& + 4A_{10} \sum_{n=0}^{\infty} \left[\operatorname{nerfc}\left(\frac{n\sqrt{P_r}}{\sqrt{t}}\right) + (n+1)\operatorname{erfc}\left(\frac{(n+1)\sqrt{P_r}}{\sqrt{t}}\right) \right] \\
& - 4A_9 \sqrt{\frac{S_c}{\pi t}} \sum_{n=0}^{\infty} \left[n^2 \exp\left(-\frac{n^2 S_c}{t}\right) + (n+1)\operatorname{erfc}\left(-\frac{(n+1)^2 S_c}{t}\right) \right] \\
& - 4A_{10} \sqrt{\frac{P_r}{\pi t}} \sum_{n=0}^{\infty} \left[n^2 \exp\left(-\frac{n^2 P_r}{t}\right) + (n+1)\operatorname{erfc}\left(-\frac{(n+1)^2 P_r}{t}\right) \right] \\
& \quad - 4A_5 \sum_{n=0}^{\infty} \left[\operatorname{nerfc}\left(\frac{n}{t}\right) + (n+1)\operatorname{erfc}\left(\frac{(n+1)}{t}\right) \right] \\
& + 4A_5 \sqrt{\frac{S_c}{\pi t}} \sum_{n=0}^{\infty} \left[n^2 \exp\left(-\frac{n^2}{t}\right) + (n+1)^2 \exp\left(-\frac{(n+1)^2}{t}\right) \right] \\
& + 2A_{11} \sqrt{t} \sum_{n=0}^{\infty} \left[n \exp\left(-\frac{n^2 S_c}{t}\right) + (n+1) \exp\left(-\frac{(n+1)^2 S_c}{t}\right) \right] \\
& - \frac{4S_c A_{11}}{\sqrt{t}} \sum_{n=0}^{\infty} \left[n^3 \exp\left(-\frac{n^2 S_c}{t}\right) + (n+1)^3 \operatorname{erfc}\left(-\frac{(n+1)^2 S_c}{t}\right) \right] \\
& + 2A_{12} \sqrt{t} \sum_{n=0}^{\infty} \left[n \exp\left(-\frac{n^2 P_r}{t}\right) + (n+1) \exp\left(-\frac{(n+1)^2 P_r}{t}\right) \right] \\
& - \frac{4A_{12} P_r}{\sqrt{t}} \sum_{n=0}^{\infty} \left[n^3 \exp\left(-\frac{n^2 P_r}{t}\right) + (n+1)^3 \exp\left(-\frac{(n+1)^2 P_r}{t}\right) \right] \\
& \quad + 2A_{13} \sum_{n=0}^{\infty} \left[n \exp\left(-\frac{n^2}{t}\right) + (n+1) \exp\left(-\frac{(n+1)^2}{t}\right) \right] \\
& \quad - \frac{4A_{13}}{\sqrt{t}} \sum_{n=0}^{\infty} \left[n^3 \exp\left(-\frac{n^2}{t}\right) + (n+1)^3 \exp\left(-\frac{(n+1)^2}{t}\right) \right] \\
& \quad + \frac{R_c A_{14}}{t\sqrt{t}} \sum_{n=0}^{\infty} \left[n^2 \exp\left(-\frac{n^2}{t}\right) + n(n+1) \exp\left(-\frac{(n+1)^2}{t}\right) \right] \\
& - \frac{R_c A_{14}}{t\sqrt{t}} \sum_{n=0}^{\infty} \left[(n+1)^2 \exp\left(-\frac{n^2}{t}\right) + (n+2)(n+1) \exp\left(-\frac{(n+1)^2}{t}\right) \right]
\end{aligned} \tag{17}$$

$$\begin{aligned}
 \tau_1 = -\frac{\partial u}{\partial y}\Big|_{y=1} = & -(tA_2 + R_cA_7)Sh_1 + 2(tA_3 + R_cA_6)\sqrt{\frac{P_r}{\pi t}} \sum_{n=0}^{\infty} [\exp(-\frac{(2n+1)^2 P_r}{t})] \\
 & - \frac{2(tA_4 + R_cA_8)}{\sqrt{\pi t}} \sum_{n=0}^{\infty} \exp(-\frac{(2n+1)^2}{t}) - 4A_9 \sum_{n=0}^{\infty} (2n+1) \operatorname{erfc}(-\frac{(2n+1)\sqrt{S_c}}{\sqrt{t}}) \\
 & - 4A_{10} \sum_{n=0}^{\infty} (2n+1) \operatorname{erfc}(\frac{(2n+1)\sqrt{P_r}}{\sqrt{t}}) + 2A_9 \sqrt{\frac{S_c}{\pi t}} \sum_{n=0}^{\infty} (2n+1)^2 \exp(-\frac{(2n+1)^2 S_c}{t}) \\
 & + 2A_{10} \sqrt{\frac{P_r}{\pi t}} \sum_{n=0}^{\infty} (2n+1) \exp(-\frac{(2n+1)^2 P_r}{t}) + 4A_5 \sum_{n=0}^{\infty} (2n+1)^2 \operatorname{erfc}(-\frac{(2n+1)}{t}) \\
 & - 2A_5 \sqrt{\frac{S_c}{\pi t}} \sum_{n=0}^{\infty} (2n+1)^2 \exp(-\frac{(2n+1)^2}{t}) - 2A_{11} \sqrt{t} \sum_{n=0}^{\infty} (2n+1) \exp(-\frac{(2n+1)^2 S_c}{t}) \\
 & + \frac{S_c A_{11}}{\sqrt{t}} \sum_{n=0}^{\infty} (2n+1)^3 \exp(-\frac{(2n+1)^3 S_c}{t}) - 2A_{12} \sqrt{t} \sum_{n=0}^{\infty} (2n+1) \exp(-\frac{(2n+1)^2 P_r}{t}) \\
 & + \frac{P_r A_{12}}{\sqrt{t}} \sum_{n=0}^{\infty} (2n+1)^3 \exp(-\frac{(2n+1)^3 P_r}{t}) - 2A_{13} \sqrt{t} \sum_{n=0}^{\infty} (2n+1) \exp(-\frac{(2n+1)^2}{t}) \\
 & + \frac{A_{13}}{\sqrt{t}} \sum_{n=0}^{\infty} (2n+1)^3 \exp(-\frac{(2n+1)^3}{t}) - \frac{R_c A_{14}}{t\sqrt{t}} \sum_{n=0}^{\infty} n(2n+1) \exp(-\frac{(2n+1)^2}{t}) \\
 & + \frac{R_c A_{14}}{t\sqrt{t}} \sum_{n=0}^{\infty} (n+1)(2n+1) \exp(-\frac{(2n+1)^2}{t})
 \end{aligned}
 \tag{18}$$

Steady Case: Setting $\frac{\partial()}{\partial t} = 0$ in the equations (6)-(8), the steady state of the problem is obtained as

$$\frac{d^2 u}{dy^2} + NC + T = 0 \tag{19}$$

$$\frac{d^2 C}{dy^2} = 0 \tag{20}$$

$$\frac{d^2 T}{dy^2} + D^* \frac{d^2 C}{dy^2} = 0 \tag{21}$$

Using equation (20) in (21), it becomes $\frac{d^2T}{dy^2} = 0$, which is independent of the diffusion-thermo effect. The solution of the equations (19)-(21) subject to boundary conditions (9) are

$$\tilde{C} = 1 - y \quad (22)$$

$$\tilde{T} = 1 - y \quad (23)$$

$$\tilde{u} = \frac{N+1}{6}(y^3 - 3y^2 + 2y) \quad (24)$$

In this case the rate of mass transfer and the skin friction are given by

$$\tilde{S}h_0 = \tilde{S}h_1 = 1 \quad (25)$$

$$\tilde{N}u_0 = \tilde{N}u_1 = 1 \quad (26)$$

$$\tilde{\tau}_0 = \frac{N+1}{3} \quad (27)$$

$$\tilde{\tau}_1 = \frac{N+1}{6} \quad (28)$$

4. Results and Discussion

The effects of pertinent parameters P_r , the Prandtl number, D^* , the Dufour number, S_c , the Schmidt number, N , the sustention parameter and R_c , the elastic parameter are exhibited through the graphs on the free convection and mass transfer of a visco-elastic fluid in a vertical channel with Dufour effect. The expressions for the velocity, temperature and mass transfer are shown graphically in figures 1 to 3 and also the skin friction coefficient is given in the table. The solution for steady case indicates that temperature curves are linear in nature.

The equation (6) represents the x-momentum equation of visco-elastic fluid of Walters \dot{B} model. From equation (6) we get

- (i) $R_c=0.0$ (without elasticity) case of Jha and Ajibade [1].

(ii) $R_c=0.0$, $D^*=0.0$ (without elasticity and diffusion thermo effect) and without mass transfer case of Jha and Singh [24].

Fig.1 shows the variation of velocity with the help of various parameters. The nature of variation is parabolic which attains the maximum value in the middle of channel. All the parameters characterizing the velocity distribution enhance it. An increase in the value of sustentation parameter N , contributes maximum to enhance the velocity in case of $P_r=0.71$. On comparing curves VII and VIII it is seen that an increase in N leads to decrease in the velocity. It is concluded that dominance of momentum diffusivity i.e higher Prandtl number (P_r) fluid reduces the velocity at all point in the flow domain. Another point of interest is the role of elastic property of the fluid. The increasing value of elastic parameter and sustentation parameter increase the velocity at all points in the flow domain except a few layer near the plate. In the absence of elasticity, $R_c=0.0$ (curve I) accelerates the appearance of pick of the profile earlier than the curve-II, $R_c \neq 0.0$.

Further, the instability is marked for low span of time. Thus the present study reveals that high Dufour effect and the low time span are not desirable for attaining the stability of flow. The consideration elasticity of the fluid is beneficial in accelerating velocity in the flow.

Fig.2 shows that the effect of temperature distribution for various values of Prandtl number, Dufour number and Schmidt number. Without considering the Dufour effect the temperature equation becomes decoupled and hence independent of concentration variation. In temperature profile P_r , exhibits the relative importance of heat conduction and viscosity of the fluid i.e the ratio of momentum diffusivity due to viscosity and thermal diffusivity due to conductivity. The significant rise in temperature is marked as the viscous diffusivity, thermal diffusivity and mass diffusivity favours each other and the longer span

of time accelerates the temperature further. An increase in sustention parameter (N) as well as Dufour number (D^*) enhance the temperature at all points of the channel. It is observed in case of heavier species (large S_c) i.e with increasing viscous diffusivity the temperature rises. Further, with large value of Dufour number (D^*), the fluid temperature rises. It is most interesting to note that an increase in time enhance the temperature.

Fig.3 Exhibits the concentration profile for various values of Schmidt number (S_c) and time (t). For aqueous medium the (S_c) takes higher values and hence it is computed for (S_c)=100 and (S_c)=617. It is found that in case of unsteady flow an increase in (S_c) leads to decrease the concentration level at all points but the reverse effect is observed in case of time span. Further, it is interesting to note that for (S_c)=1.0 the concentration level is high in comparison with the case of heavier species (S_c)=100 and (S_c)=617, and it increases with the longer span of time.

5. Skin Friction

From table.1 it is observed that the skin friction at lower plate decreases in magnitude whereas at the upper plate it increases in the presences of visco-elastic element in the fluid but at the time span reverse effect is observed. This shows that the characteristic of elasticity in the flow behavior depends upon time span. The table further reveals that high Prandtl number fluid causes reduction in the skin friction which is desirable. It is interesting to note that Sustention parameter, Dufour number and Schmidt number increases the skin friction at both the plate. Thus, the presence of heavier species associated with these two effects results in a counterproductive yielding higher rate of skin friction.

6. Conclusion

- (i) Higher Prandtl number fluid reduces the velocity at all point

in the flow domain.

- (ii) The increasing value of elastic parameter and Suction parameter increase the velocity at all points in the flow domain.
- (iii) High Dufour effect and the low time span are not desirable for attaining the stability of flow.
- (iv) Elasticity of the fluid accelerates the velocity in the flow.
- (v) The longer span of time accelerates the temperature.
- (vi) Increase in sustentation parameter (N) as well as Dufour number (D^*) enhance the temperature at all points of the channel.
- (vi) Increases in Schmidt number (S_c) leads to decrease the concentration level at all points but the reverse effect is observed in case of time span.
- (vii) Elasticity of the fluid decreases the skin friction at lower plate where as increases the skin friction at the upper plate but the reverse is observed in case of time span.
- (viii) High Prandtl number fluid reduces the skin friction where as sustentation parameter, Dufour number and Schmidt number increases the skin friction at both the plate.

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Appendix:

$$A_1 = \frac{D^*S_c}{P_r - S_c},$$

$$A_2 = \frac{N + A_1}{1 - S_c},$$

$$A_3 = \frac{1 - A_1}{1 - P_r},$$

$$A_4 = A_2 + A_3,$$

$$A_5 = \frac{A_4}{2},$$

$$A_6 = \frac{1 - A_1 - A_3}{1 - P_r},$$

$$A_7 = \frac{N + A_1 - A_3}{1 - S_c},$$

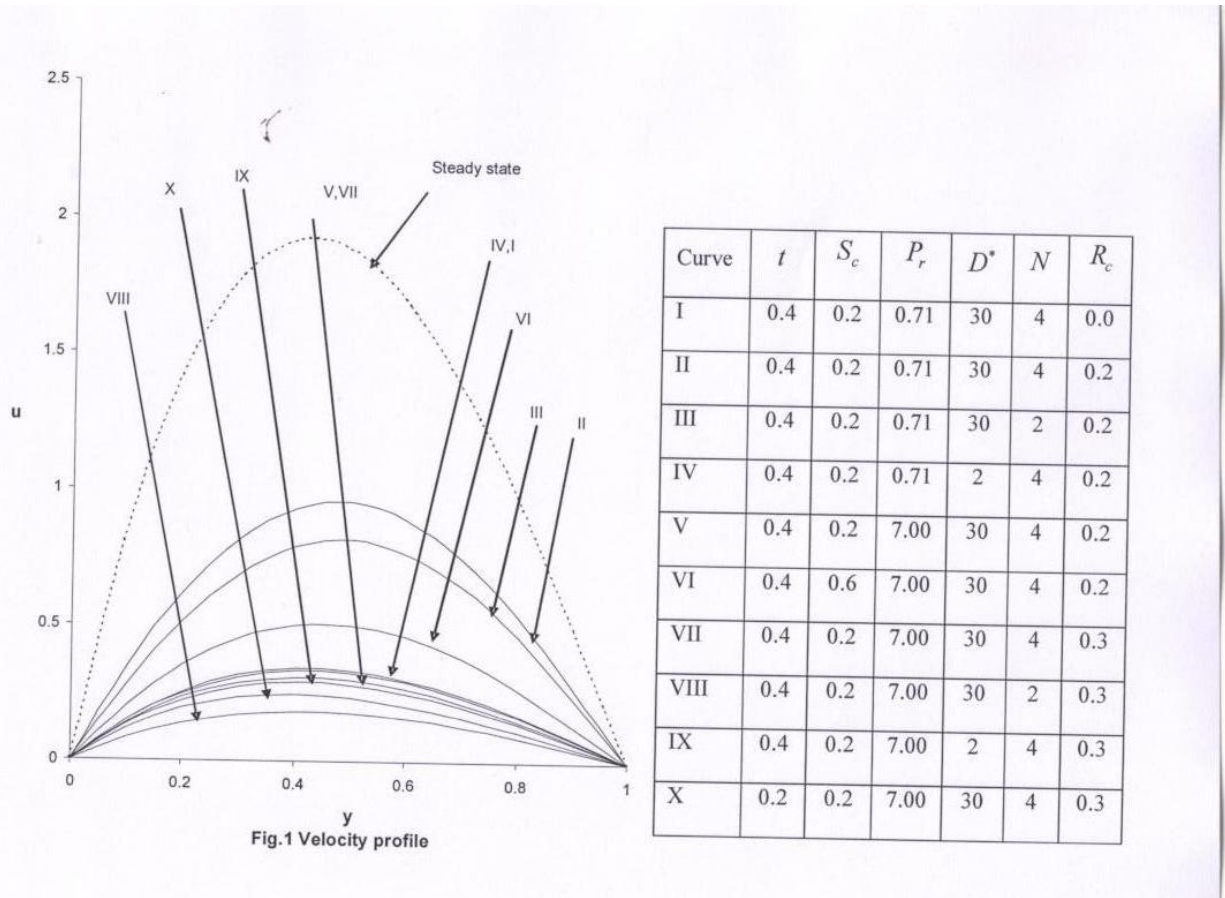
$$A_8 = A_6 + A_7,$$

$$A_9 = \frac{A_2S_c}{2},$$

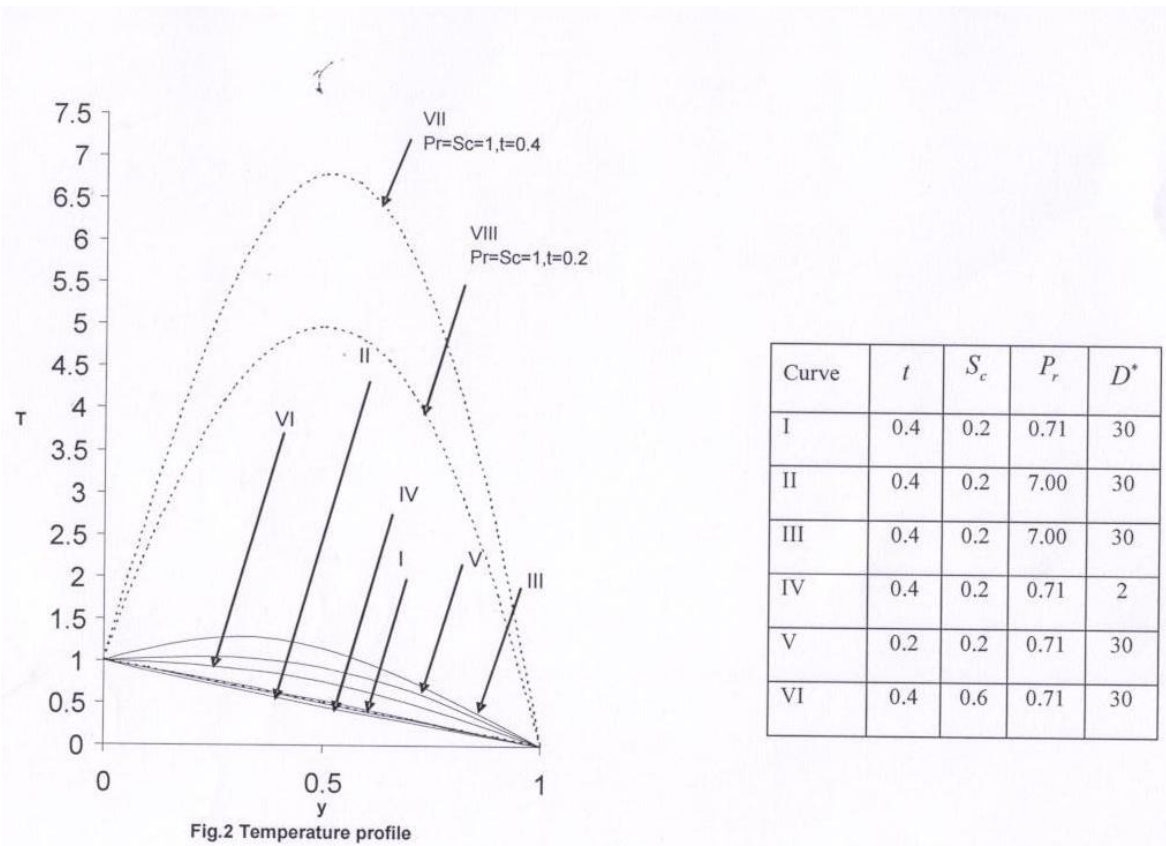
$$A_{10} = \frac{A_3P_r}{2},$$

$$A_{11} = -\frac{A_2\sqrt{S_c}}{\sqrt{\pi}},$$

$$A_{12} = -\frac{A_3\sqrt{P_r}}{\sqrt{\pi}}, A_{13} = \frac{A_4}{\sqrt{\pi}}, A_{14} = \frac{A_5}{\sqrt{\pi}}$$



Curve	t	S_c	P_r	D^*	N	R_c
I	0.4	0.2	0.71	30	4	0.0
II	0.4	0.2	0.71	30	4	0.2
III	0.4	0.2	0.71	30	2	0.2
IV	0.4	0.2	0.71	2	4	0.2
V	0.4	0.2	7.00	30	4	0.2
VI	0.4	0.6	7.00	30	4	0.2
VII	0.4	0.2	7.00	30	4	0.3
VIII	0.4	0.2	7.00	30	2	0.3
IX	0.4	0.2	7.00	2	4	0.3
X	0.2	0.2	7.00	30	4	0.3



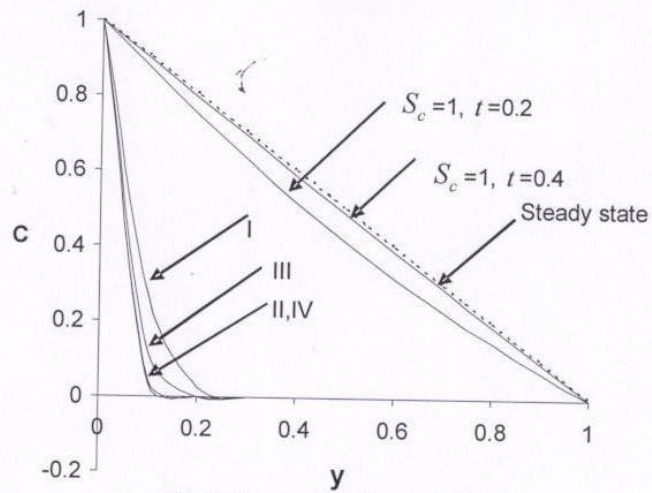


Fig.3 Concentration profile

Curve	t	S_c
I	0.4	100
II	0.4	617
III	0.2	100
IV	0.2	617

Table: 1 Value of Skin Friction at both the lower and upper plate

t	S_c	P_r	D^*	N	R_c	τ_0	τ_1
0.4	0.2	0.71	30	4	0.0	-22.89840479	10.98813751
0.4	0.2	0.71	30	4	0.2	-8.465112063	20.42784769
0.4	0.2	0.71	30	2	0.2	-7.224136571	18.93147876
0.4	0.2	0.71	2	4	0.2	-2.99498901	3.406575923
0.4	0.2	7.00	30	4	0.2	-7.999530903	3.658187842
0.4	0.6	7.00	30	4	0.2	-29.54603282	9.017302369
0.4	0.2	7.00	30	4	0.3	-9.261732298	4.214269272
0.4	0.2	7.00	30	2	0.3	-5.478277444	2.491026126
0.4	0.2	7.00	2	4	0.3	-7.883398439	3.557399103
0.2	0.2	7.00	30	4	0.3	-11.15702451	3.213176252